Entanglement distillation from Gaussian input states

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(Dated: December 21, 2009)

Entanglement distillation is an essential protocol for long distance quantum communications [1], typically for extending the range of quantum key distribution (QKD). In the field of continuous variable quantum information processing (CV-QIP), quantum as well as classical information is encoded in the light field quadratures, often in the form of Gaussian states. However, distillation from Gaussian input states has not yet been accomplished. It is made difficult by a prominent no-go theorem stating that no Gaussian operation can distill Gaussian states [2–4]. Here we demonstrate for the first time such distillation from Gaussian input states, realized by the implementation of non-Gaussian operations: By subtracting one or two photons, a large gain of entanglement was observed. For two photons, also Gaussian-like entanglement was improved. Apart from QKD, this distilled entanglement can furthermore be employed to downstream applications such as high fidelity quantum teleportation [5] and a loophole-free Bell test [6, 7].

Long distance quantum communications rely on the ability to faithfully distribute entanglement between distant locations. However, inevitable decoherence and the inability to amplify quantum signals hinder efforts to extend a quantum optical link to a practically large scale. To overcome this problem, entanglement distillation can be used - a protocol in which each distant party locally manipulates particles of less entangled pairs with the aid of classical communication to extract a smaller number of pairs of higher entanglement [8].

Many distillation experiments have already been demonstrated in discrete variable systems (e.g. qubits) [9–12], but the situation is different within CV-QIP, the other major realm of quantum information. As is the case with certain kinds of advanced optical communications technology, information is encoded in the quadrature amplitude and phase of a light field, and can be decoded by homodyne detection. The carrier of classical information is typically the coherent state and non-classical resources for QIP can be provided by squeezed and two-mode squeezed (entangled) states. These states are called Gaussian states since they all exhibit Gaussian statistics in the quadratures. Gaussian states and Gaussian operations - quantum operations preserving the Gaussian nature of a state - combine a simple theoretical description with straight-forward experimental implementation to provide a complete framework for many QIP protocols, e.g. teleportation or dense coding [13, 14]. In spite of the importance of Gaussian states, distillation of entanglement from them has not been achieved so far. That is due to the difficulties imposed by the no-go theorem, stating that Gaussian operations alone can never distill entanglement from Gaussian states [2–4]. Recently, two experiments of CV distillation were reported. The trick there was that the inputs had been subjected to some specific types of non-Gaussian noise, namely phase-diffusion [15] and temporally varying attenuation [16]. In those cases, well established Gaussian processes could be applied to distill the decohered entanglement.

In the more general case of Gaussian input states, though, a new technology of quantum nonlinear processes - non-Gaussian operation - is needed. More broadly, recent theories also revealed that this is a must to realize quantum speed-up of CV QIP [17]. Triggered by this new paradigm of non-Gaussian QIP, the research field extending to the non-Gaussian regime has rapidly developed [18–20]. Two important results by Ourjoumtsev et al. demonstrated the increase [21] and preparation [22] of entanglement from Gaussian inputs, and thus proved that photon subtraction is capable of providing the nonlinearity needed for distillation. The experiments did not show true distillation, though, since the operation was performed *non-locally*. Furthermore, the two entangled modes were not spatially separated.

Here we report on the entanglement distillation directly from CV Gaussian states by using local photon subtraction as non-Gaussian operations, thereby circumventing the no-go restriction on Gaussian operations. A schematic of our experiment is depicted in Fig. 1. A continuous wave squeezed vacuum is generated from an optical parametric oscillator (OPO) detailed elsewhere [20]. The initial Gaussian entangled state is prepared by splitting the squeezed vacuum by half at the first beam splitter and is distributed to the separate parties, Alice and Bob. This half-split squeezed vacuum with squeezing parameter r is effectively equivalent to the two-mode squeezed vacuum with $r/2$ (they are compatible by local unitary operations – see Supplementary Information

I.A). At each site of Alice and Bob, a probabilistic non-Gaussian operation – photon subtraction – is performed. Specifically, a small part of the beam is picked off by a polarizing beam splitter with the variable reflectance R and sent through filtering cavities [20] to an avalanche photodiode (APD) to detect a photon. Each photon detection at the APD heralds a local success of the photon subtraction attempt. Conditioned on the subtraction of a photon by a single party (single-photon subtraction) or the simultaneous subtraction of a photon by both parties (two-photon subtraction), Alice and Bob retain those two-mode states which have successfully had their entanglement increased. While the single-photon subtraction scheme will have a higher success rate, the two-photon subtraction scheme will give a more Gaussian-like final state which is more readily applicable to further processing such as e.g. quantum teleportation.

The distillation works since the local photon subtraction changes the non-local unfactorizable correlations of the initial state. To see this intuitively, let us describe the initial squeezed state with squeezing parameter r in photon number basis as $|0,0\rangle - \frac{\lambda}{2\sqrt{2}}(|0,2\rangle + \sqrt{2}\,|1,1\rangle + |2,0\rangle) + O(\lambda^2)$ where $|m,n\rangle = |m\rangle_A|n\rangle_B$, $\lambda = \tanh r$ and we omitted the normalization. For small λ, it is almost factorizable since $|0,0\rangle$ is dominant. Applying the single-photon subtraction, represented by an annihilation operator $\hat{a}$, the state is transformed to be $\sqrt{2}(|0,1\rangle + |1,0\rangle) + O(\lambda)$ which is clearly more entangled - the first Bell state term corresponds to 1 ebit of entanglement. We emphasize that the term $O(\lambda)$ is not negligible for larger λ and critically contributes to the distillation, and thus the scheme works for any λ in principle. See Supplementary Information I.B for the rigorous formulation and for the two-photon subtraction.

The verification of the distilled (or undistilled) states is carried out by a quantum tomographic method with two local homodyne measurements (see Fig. 1). For tomography, in general the local oscillator (LO) phases θ_A and θ_B (for Alice and Bob's detectors, respectively) have to be swept over all possible combinations to collect full information of the two-mode state. In our case, however, it can be significantly simplified due to the fact that one of the inputs on the initial half-beam splitter is vacuum. That means only components originating from the squeezed vacuum is affected by the photon subtraction, so via an inverse beam splitter transformation, our states, distilled or undistilled, can always be represented in a decomposed way as

$$W(x_A, p_A, x_B, p_B) = W_s(x_-, p_-)W_v(x_+, p_+), \qquad (1)$$

where W is the Wigner function for the two-mode state, x_A, x_B, p_A, p_B are the quadrature amplitudes of modes A and B, $x_\pm = \frac{x_A \pm x_B}{\sqrt{2}}$, $p_\pm = \frac{p_A \pm p_B}{\sqrt{2}}$ and W_s, W_v are the Wigner functions for a (zero-, one-, or two-) photon subtracted squeezed vacuum and the vacuum respectively.

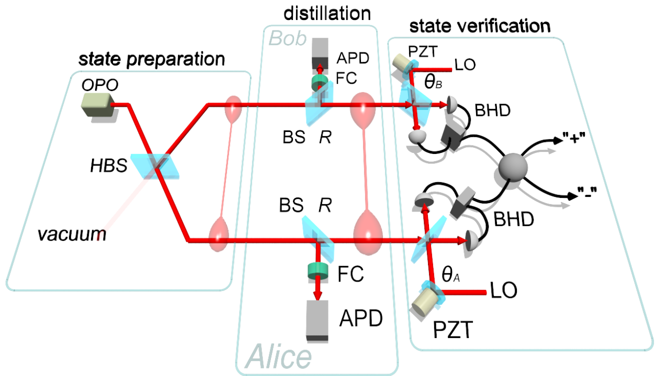

FIG. 1: **Schematic of the experiment.** OPO: optical parametric oscillator, BS: beam splitter, HBS: half-beam splitter, BHD: balanced homodyne detector, LO: local oscillator, FC: filtering cavity, APD: avalanche photo diode, PZT: Piezo electric transducer. The phases of the LOs, θ_A and θ_B are stabilized by electronic feedback to the PZTs through field programmable gate array (FPGA) modules. The outputs of the BHDs are recorded by a digital oscilloscope triggered by either logical "AND" or "OR" of the click signals from the two APDs. For the state verification, a set of the homodyne outcomes are numerically converted into the "+/−" basis.

For such a state, the scans of the homodyne measurements are necessary only for $\theta_A = \theta_B$ and the experimental data of $x_\pm$ is numerically obtainable from the measured x_A and x_B (Fig. 1). It should be stressed that although we assume the state factorization of (1), it can be directly assessed by verifying experimentally whether the state of the "+" mode is indeed a pure vacuum state. For details, see Supplementary Information I.C.

Examples of reconstructed Wigner functions obtained by the single- and two-photon subtractions, as well as the initial squeezed state are shown in Fig. 2a-c. The outputs of the homodyne detectors were sampled at 6 different phases of LO, namely $\theta_A = \theta_B = 0, \pi/6, \pi/3, \pi/2, 2\pi/3, 5\pi/6$. We extract the measured values of the quadratures $\hat{x}_A$ and $\hat{x}_B$ by applying a mode function to the recorded traces [19, 20, 23]. After calculating the corresponding values of $\hat{x}_\pm$, we reconstruct the density matrices for the "+" and "−" modes by the conventional maximum likelihood estimation [24] without any correction of detection losses.

As shown in Fig. 2e, for the "+" mode states we got almost perfectly pure vacuum states with more than 99% accuracy. We confirmed that this holds irrespective of the initial squeezing level. This experimental evidence justifies our tomography scheme based on the relation (1). On the other hand, for the "−" mode states we observed two different kinds of non-Gaussian state depending on whether single photon or two photons were subtracted (Fig. 2a and b). They respectively correspond to the odd and even Schrödinger cat state, i.e. $|\alpha\rangle - |-\alpha\rangle$ and $|\alpha\rangle + |-\alpha\rangle$ where $|\alpha\rangle$ is a coherent state with coherent

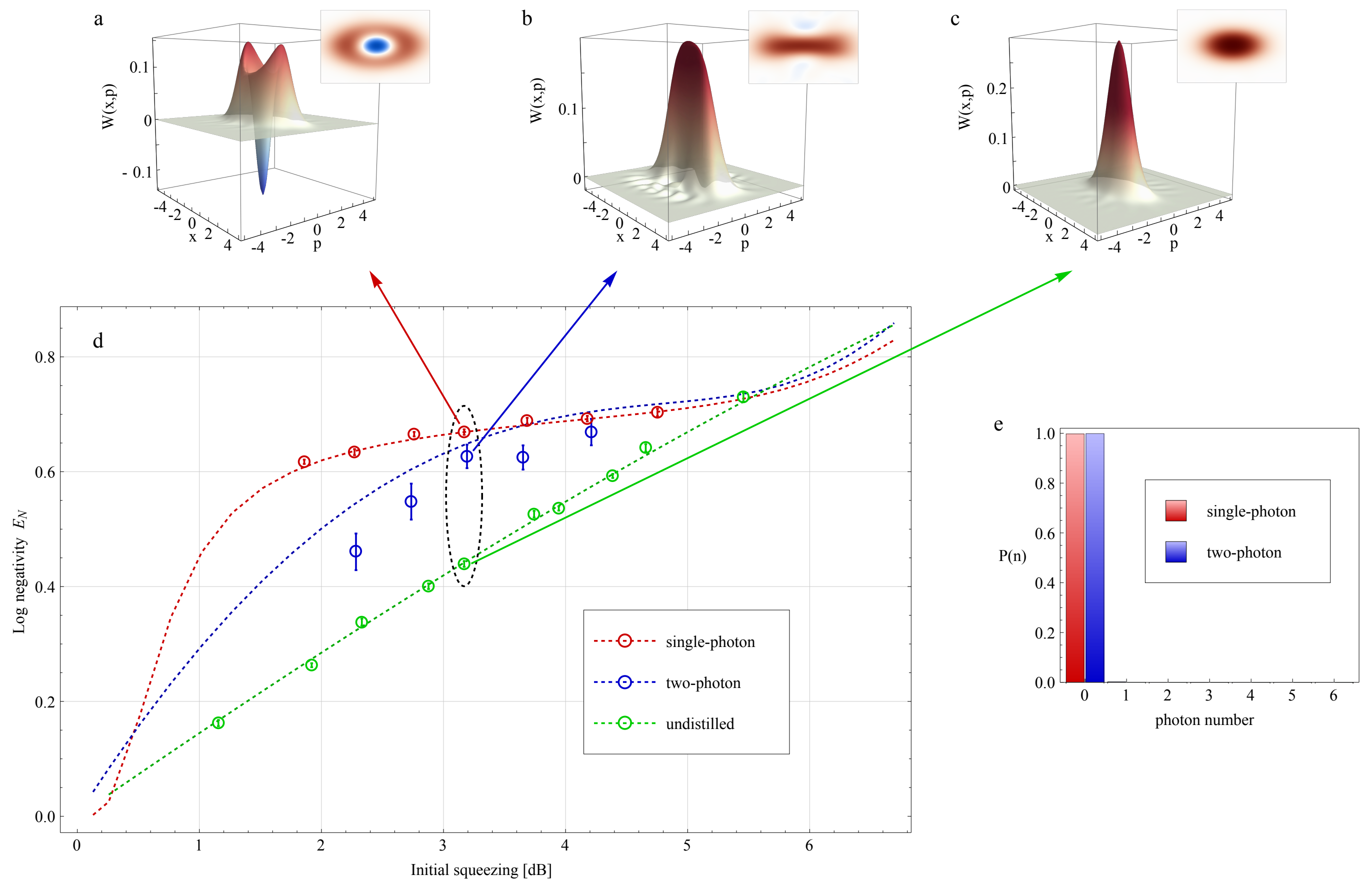

FIG. 2: **Distillation results. a, b, c** Experimentally reconstructed Wigner functions and their contour plots of the "−" mode states for (a) Distilled state via single-photon subtraction with R =5%, (b) Distilled state via two-photon subtraction with R =10%, and (c) Undistilled initial state (squeezed vacuum with R =0%), all with initial squeezing of -3.2 dB. **d** Experimental logarithmic negativities as functions of the initial input squeezing. Here the initial squeezing refers to the squeezing of the states right after the OPO and is deduced from separately measured classical parametric amplification of the OPO. For the single-photon subtracted states (red) and the undistilled states (green), 600,000 samples over 6 phases were used for the reconstruction of each point. For the two-photon subtracted states (blue) 18,000 - 48,000 samples were used. The dashed curves are theoretical predictions based on independently measured experimental parameters. Every error bar represents an uncertainty of the state reconstruction and was estimated via a MonteCarlo simulation using the corresponding experimental parameters. **e**, The photon number distributions of the experimentally reconstructed "+" mode states corresponding to (a) and (b).

amplitude α. Having these reconstructed states we can use the relation (1) backwards to calculate the amount of entanglement shared by Alice and Bob. Specifically, we calculate the logarithmic negativity $E_\mathcal{N}$ which is a monotone measure of entanglement [25].

Fig. 2d shows the experimental negativities of the undistilled Gaussian states, the states distilled by single-photon subtraction with $R = 5\%$, and by two-photon subtraction with $R = 10\%$ as functions of the squeezing of the initial input states. When evaluating negativity, one must take care of its strong dependency on the size of the data set. We investigated the behavior of the negativity on the data size and deduced an extrapolative value corresponding to an infinitely large data set for each point in Fig. 2d (See Supplementary Information II). Note that without this analysis, evaluation of negativity with finite sized data very likely goes into an overestimate of the negativity. As shown in the figure, over a wide range of the initial squeezing we got clear gains of entanglement relative to the undistilled Gaussian states for both the single- and two-photon subtracted schemes.

A practical difference between the single- and two-photon subtracted scheme is on their rates of event detection. In the single-photon experiment the rate is around a few thousands per second, but in the two-photon experiment there are only a few events per second. So while for the former we can use hundreds of thousands of samples for the state reconstruction, for the latter we can only use a few tens of thousands limited by the long-term stability of the setup. In Fig. 2d the experimental neg-

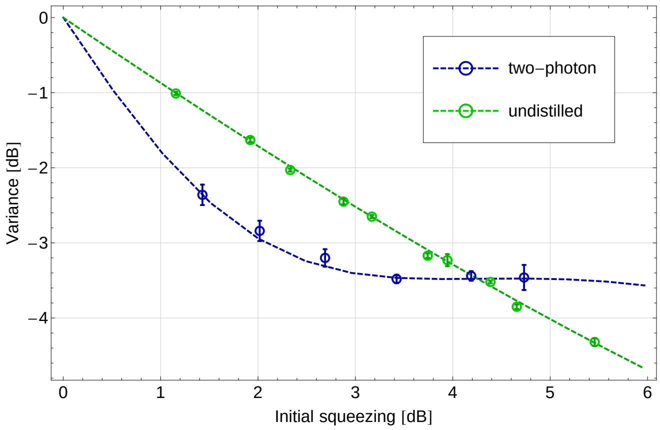

FIG. 3: **Squeezed variances of x_-.** Normalized by the vacuum level. For the undistilled states each point was calculated from the full-reconstructed density matrices common to the negativity measurements. For the distilled states each point was obtained by directly measuring variance of 1,600 - 5,000 samples at the most squeezed phase. The horizontal axis in this figure, as well as in Fig. 2, is the squeezing level right outside the OPO (inferred from the independently measured pump level and losses of the OPO), i.e., before any degradation due to propagation or inefficient detection. The data points incorporate these inefficiencies, which is why the undistilled variances show lower squeezing than their respective initial values.

atives for the single-photon subtracted states and the undistilled states are in very good agreement with theory, but ones for the two-photon subtraction are slightly below the theoretical predictions. This may be due to an uncontrollable drift of the system during a long period of the measurements.

As can be seen in Fig. 2d, in terms of the logarithmic negativity the two-photon subtracted scheme does not have an advantage over the single-photon subtracted scheme despite its significantly lower success rate. However the two-photon subtracted distillation transforms a two-mode Gaussian state into one relatively close to a Gaussian state (see Fig. 2b). Hence one would expect that states distilled by this scheme still possess a Gaussian-like property of entanglement. For Gaussian states, two-mode entanglement is usually specified in terms of the Einstein-Podolsky-Rosen (EPR) correlation quantified by $\langle(\Delta\hat{x}_-)^2\rangle\langle(\Delta\hat{p}_+)^2\rangle$. Since the "+" mode is always a vacuum state (see Eq. 1), we can focus on the degree of squeezing of the "−" mode as an equivalent measure. We carried out measurements of the variance of $\hat{x}_-$ at its most squeezed phase conditioned on two-photon subtraction (Fig. 3). Note that this measurement is considerably faster than reconstruction of a full state and is possibly more accurate due to its simplicity. The results in Fig. 3 show that the two-photon subtraction improves the EPR correlation of a state with up to 4 dB of initial squeezing. An improvement on the EPR correlation gives us an operational measure of success of the two-photon subtracted distillation in terms of the fidelity of CV quantum teleportation [5]. On the other hand, the single-photon subtracted distillation is never able to improve the degree of the EPR correlation. This fact demonstrates a clearly different nature of the distilled entanglement between the two schemes.

In conclusion, we have for the first time demonstrated CV entanglement distillation from Gaussian states by conditional local photon subtraction. Because of the importance of Gaussian states and the no-go theorem of the Gaussian distillation, this has been a long-standing experimental milestone to be achieved. Our scheme would serve as the de-Gaussifying process of a more generic distillation protocol proposed in [26] by combining it with the already demonstrated Gaussification processes [15, 16], which would realize long-distance CV quantum communications. Finally, our non-Gaussian entangled states are not only useful for communications but also for fundamental problems such as a loophole-free test of Bell's inequality [6, 7].

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Acknowledgments H.T. acknowledges the financial support from G-COE program commissioned by the MEXT of Japan.

Author contributions H.T., M.Takeoka, A.F. and M.S. conceived the project. H.T., J.S.N-N. and M.Takeuchi carried out the experiment with assistance from K.H., and H.T., J.S.N-N. and M.Takeoka analyzed the data. H.T., M.S., M.Takeoka and J.S.N-N. wrote the paper with discussions and input from all authors. The project was directed by M.S.

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